Microsimulation Framework for Urban Price-Taker Markets

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Markets in Transportation and Land Use

- Important dimension in the Integrated Urban Systems modelling
  - UrbanSim, ILUTE
- Involve interactions between two behaviourally different agents
  - Buyer and Seller
  - Mostly resulting in a successful exchange of one good at a certain value (monetary or nonmonetary)
- Examples
  - Airline Seats Auction
  - Freight Spot Market
  - Rental Market

Markets in Transportation and Land Use

- Market Classification
  - **Price-Formation Markets**
    - Sellers decide a price/value that they expect from the good
      - This acts as the starting point for a negotiation process
    - Buyers look for the available choices
    - Buyers bid for the best available products
      - The iterative process may result in success of a single bid
      - The successful bidder takes the good at a transaction price/value
  - Problem definition: "Who gets what, at what price/ value"
    - Price/value is endogenously determined
  - Examples: Owner Occupied Housing Market

Markets in Transportation and Land Use

- Market Classification
  - **Price-Taker Markets**
    - Sellers decide a fixed price/value that they want from the product/service, they are selling
    - Buyers look for the available choices
    - Buyers choose the best available product
      - Accepts the listed price/value to complete the transaction
  - Problem definition: “Who gets what”
    - Price/value of the product is the outcome of an exogenous model/process
  - Examples
    - Spot Freight Market, Rental Market

Markets in Transportation and Land Use

- Market Classification
  - **Price-Taker Markets**
    - *Combinatorial Optimization based matching process*
  - **Price-Formation Markets**

Price-Taker Markets: Structure

- Agents in the Microsimulation Market
  - 'N' Consumers
    - Choices available (0 or more), in terms of products
      - Coming out of a search process
    - Interested in attaining one product out of available choices, so as to maximize the utility or satisfy a need
  - 'M' Producers/Products
    - Differentiable from each other
    - 0 or more consumers interested in the producer/product
      - In some markets, here too, a search process will be involved (for instance, Marriage market)
    - Interested in attaining an association to one consumer (out of available consumers) so as to maximize the value/utility

Price-Taker Markets: Structure

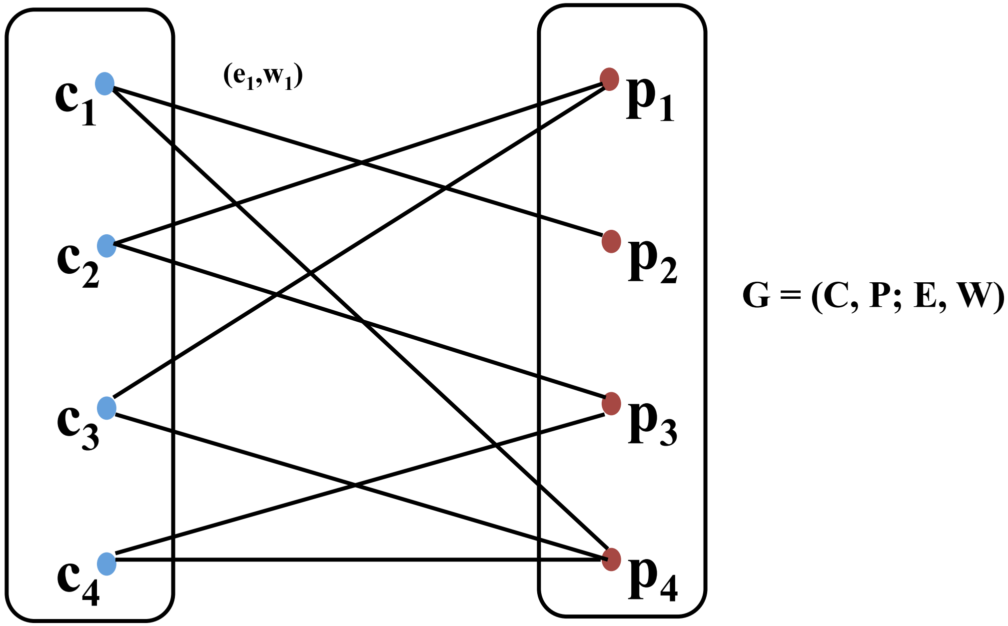
- Utility/Value of association
  - Every potential association in the choice set for both consumers and producers/products will have some utility/value associated with it
  - Might be unidirectional
    - Spot Markets
  - Or bidirectional
    - For instance in marriage market
      - Male and female associate certain utility to the choices they have
      - The solution here should satisfy the mutual utility maximization

Price-Taker Markets: Structure

- Market Clearing: Generalized Problem Definition
  - Match a single consumer with a single product/ producer under certain objective
    - For instance, the utility/value between them is maximum among the available choices they have

Price-Taker Markets

- Generalized Bipartite Graph (G) Formulation



- Consumer (C) → Blue vertices (c_1, c_2,…,c_n)
- Producers/Product (P) → Red vertices (p_1, p_2,…,p_m)
- Choices-Utility (E-W) → Edge-weights (e_1, e_2,…,e_k)
    - Unidirectional or bidirectional

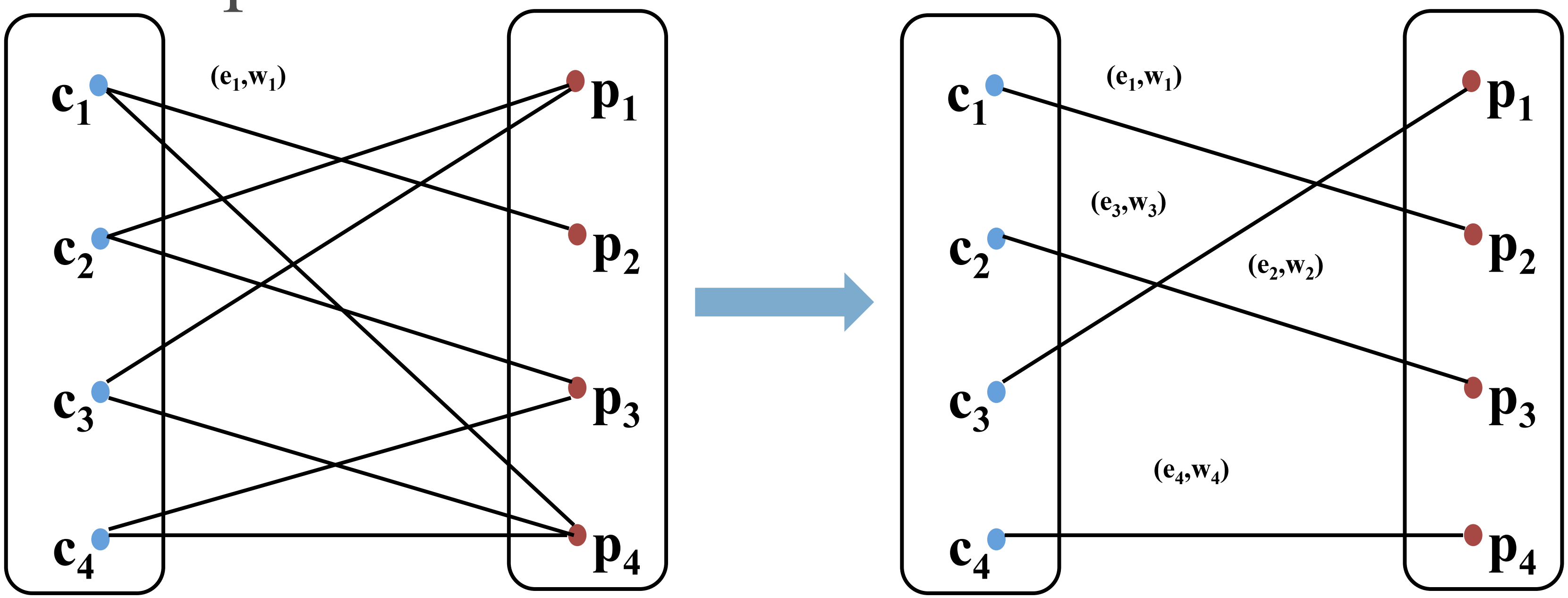
Price-Taker Markets

- Problem Redefinition

Price Taker Market Clearing ⟷ **Finding Maximum Weighted Bipartite Graph**

Price-Taker Markets

- Graph Theoretic Problem



$G = (C, P; E, W)$ $G^* = (C, P; E^*, W^*)$

$$\forall G^{**} = (C, P; E^{**}, W^{**}) : \sum_{c \in C} W^*(c, e(c)) \geq \sum_{c \in C} W^{**}(c, e(c))$$

Where, G^{**} is any potential solution and $e: C \rightarrow P$

Price-Taker Markets

- Finding Maximum Weighted Bipartite Graph
  - Special case of minimum cost flow problems
  - Solution using linear programming algorithms (Burkard et al., 2009)
    - Hungarian Algorithm (Kuhn, 1955)
    - Gaussian Elimination Algorithm (Mucha and Sankowiski, 2004)
    - Max-Product Belief Propagation Algorithm (Bayati et al., 2005)
    - Iterative Message Passing Algorithm (Cheng et al., 2006)

Price-Taker Markets

- Finding Maximum Weighted Bipartite Graph
  - Special case of minimum cost flow problems
  - Solution using linear programming algorithms (Burkard et al., 2009)
    - **Hungarian Algorithm (Kuhn, 1955)**
    - Gaussian Elimination Algorithm (Mucha and Sankowiski, 2004)
    - Max-Product Belief Propagation Algorithm (Bayati et al., 2005)
    - Iterative Message Passing Algorithm (Cheng et al., 2006)

Hungarian Algorithm

Step 0: Transform the problem into minimization problem
Step 1: For each row, subtract off the minimum cell value from rest of the cells
Each row will have at least one zero and all the values will be greater than or equal to zero
Step 2: For each column, subtract off the minimum cell value from rest of the cells
Each row and column will have thus at least one zero
Step 3: Go through rows and columns and use lines to cover the zeros in the matrix, in such a way that all the zeros are covered and that no more lines have been drawn than necessary
Use horizontal line for row and vertical for column
Step 4: Optimality test:
i. If the number of the lines is n, choose a combination from the modified matrix in such a way that the sum is zero
ii. If the number of the lines is < n, go to step 5
Step 5: Find the smallest element which is not covered by any of the lines. Then subtract it from each entry which is not covered by the lines and add it to each entry which is covered by a vertical and a horizontal line. Go back to 3

Price-Taker Markets

- Large Scale Markets
  - Hungarian Algorithm (HA) requires manipulating large size matrix
    - Order of complexity is $O(n^3)$ (Tomizawa, 1971)
    - Spare matrix could be converted into efficient lists
      - But, HA becomes computationally more expensive with the decrease in sparsity and increase in the size of market
    - HA finds a global optimal
      - That may not represent the behaviour of the agents and natural stochasticity of some price-taker markets

Efficient and Optimal Solution for Large Scale Markets

- Algorithm for finding G* = (C, P; E*, W*): **Probabilistic Approach**

  Step 1: With a predefined random distribution, pick between set C or P

  Step 2: From the selected set, choose a vertex v_1 using another predefined random distribution

  Step 3: For v_1 choose v_2 such that $w_{12} \geq w_{1i} \forall V \rightarrow v_1$, where V is the set that was not chosen in Step 1

  Step 4: Remove v_1 and v_2 and all the edges associated with them

  Step 5: Stop if either C or P becomes null set
  Else, go to Step 1

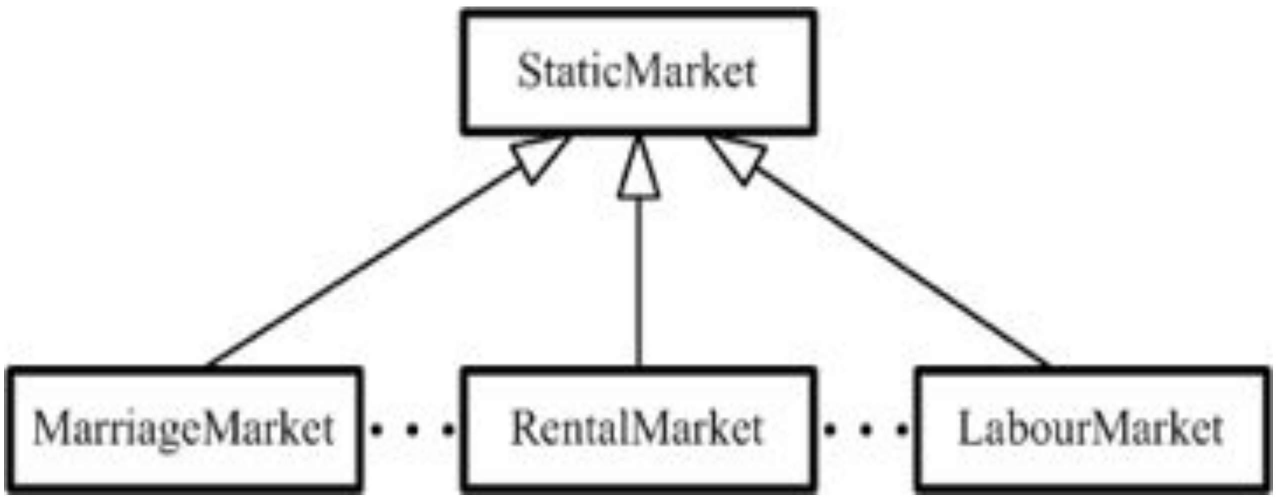
Efficient and Optimal Solution for Large Scale Markets

- Probabilistic Approach
  - Order of complexity: $O(max\ (|C|, |P|))$
    - Linear as compared to cubic in the case of Hungarian
    - Captures the natural stochasticity of the markets
      - In most cases, clearing doesn't result in global optima
    - Easy to implement in the case of large scale urban system simulations

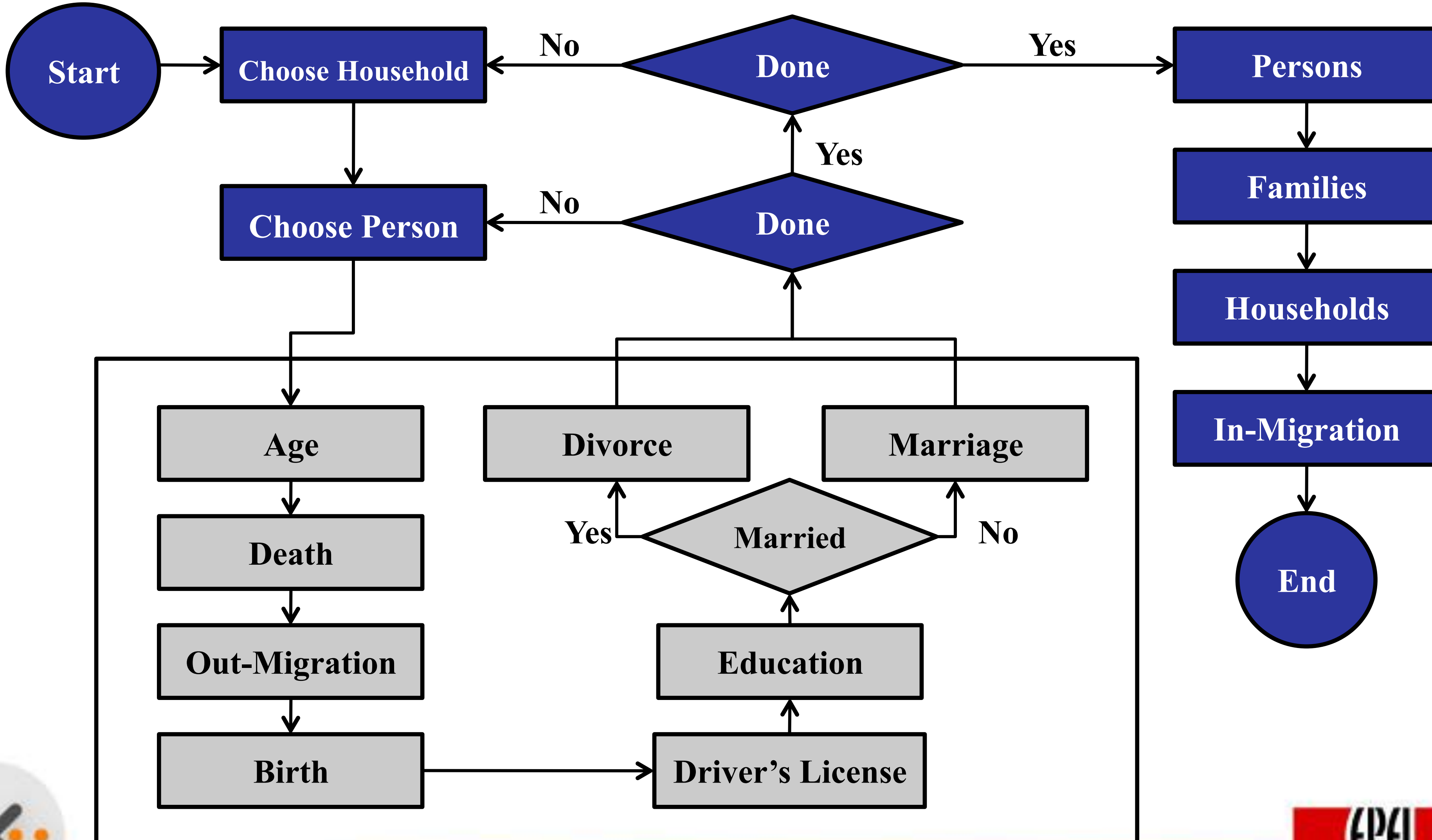
Price-Taker Markets: Application

- Integrated Land Use, Transportation and Environment (ILUTE) Urban Modelling Systems
  - Demographic Evolution: Marriage Market
  - Rental Market
  - *Labour Market*

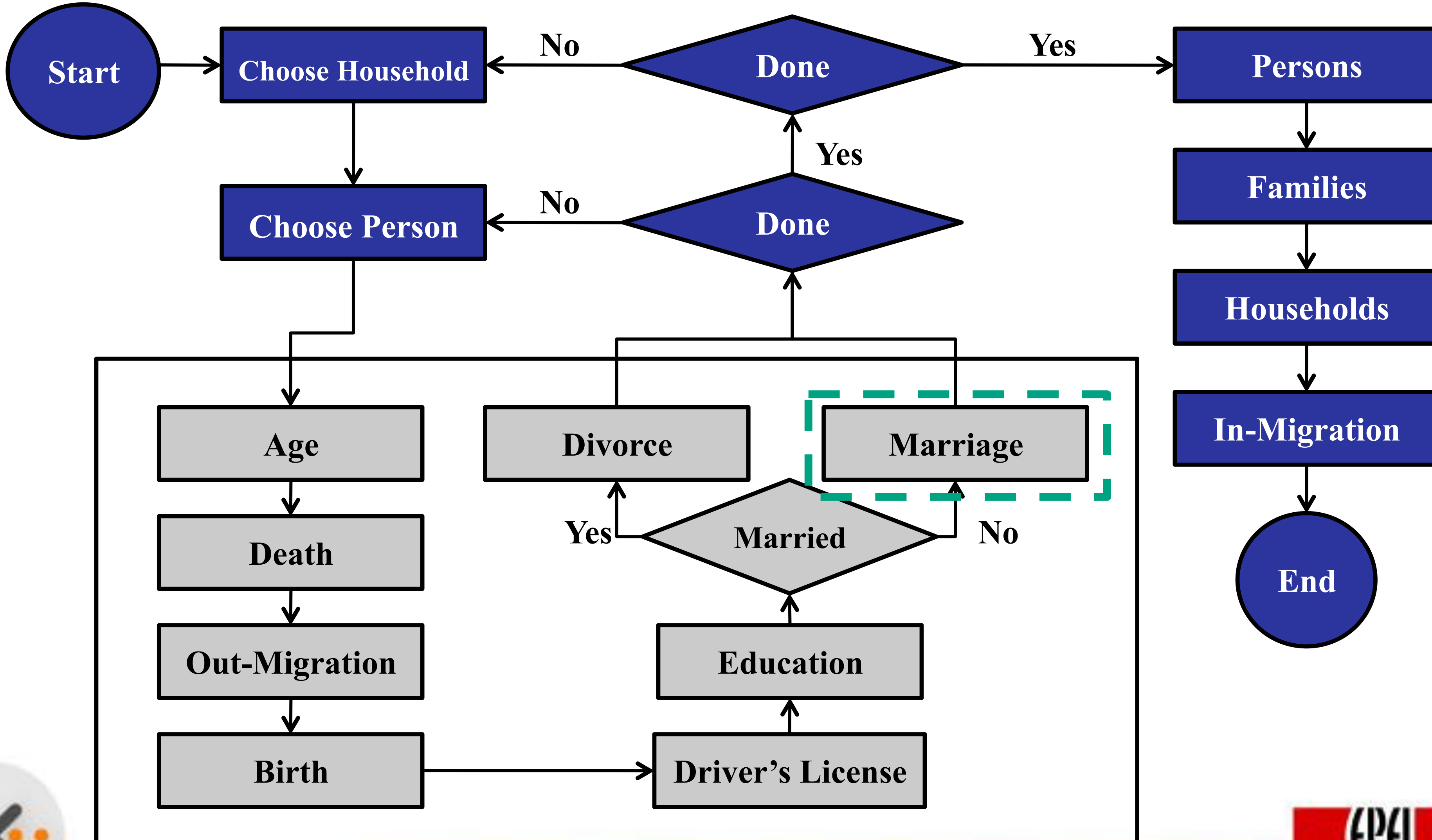
Price-Taker Markets: Application



Demographics in ILUTE: Process Flow

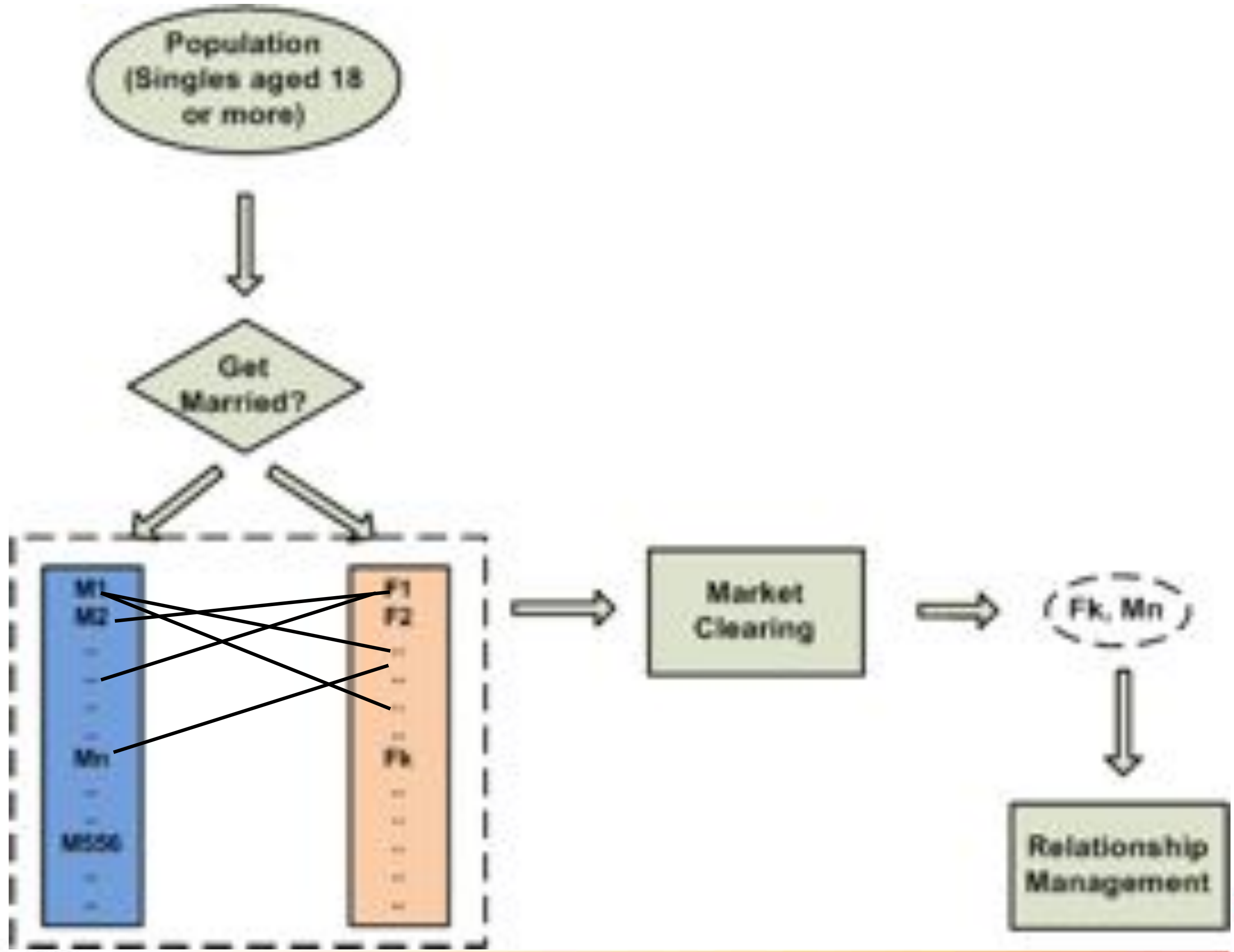


Demographics in ILUTE: Process Flow



Marriage Markets

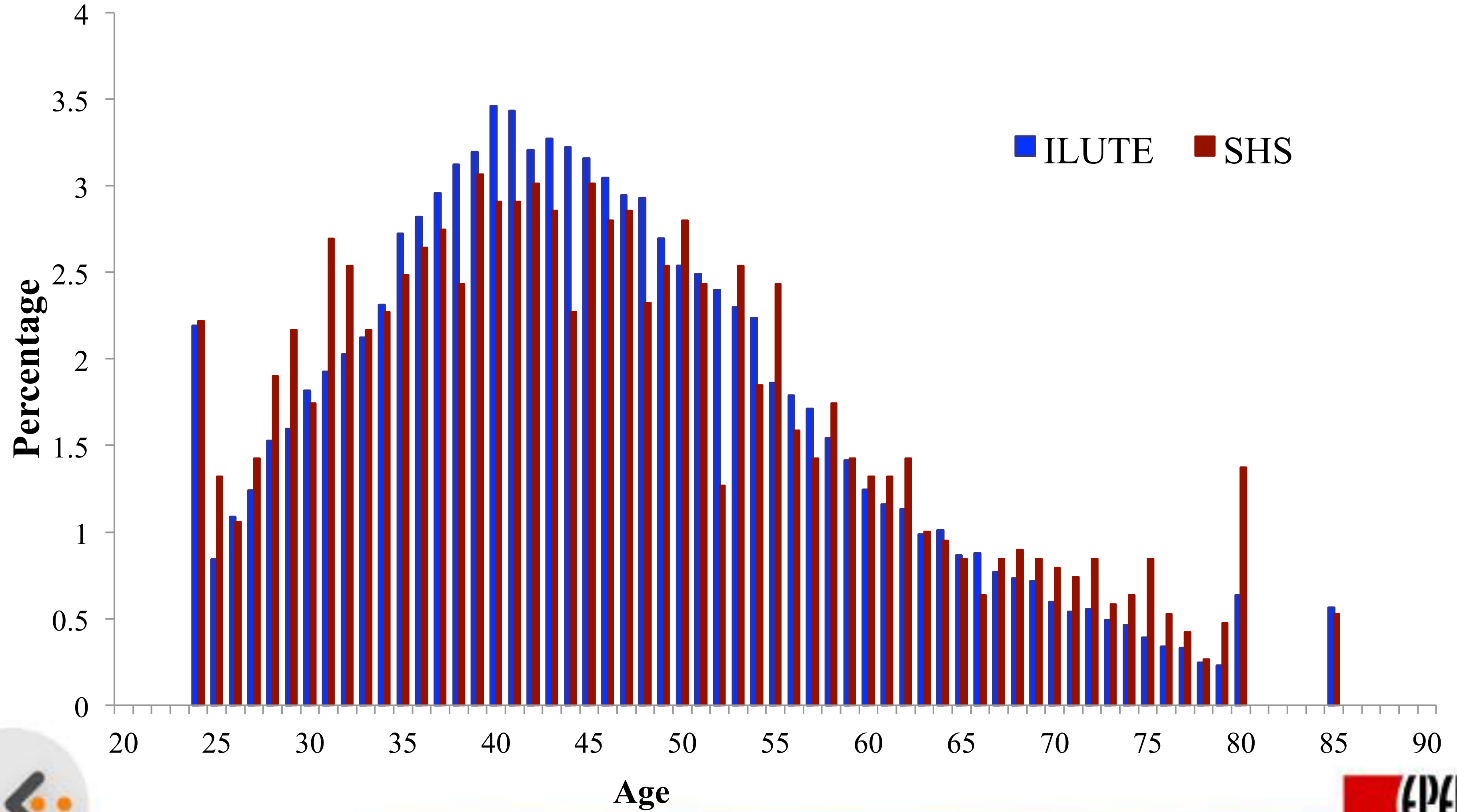
- Choice set Generation
- Utility Calculation



ILUTE Simulation Runs

- Monte Carlo simulation on 10% to 100% sample
- Historic validation
  - 1986 to 2006
  - 20 runs averaging
- C++ based native implementation
  - Runtimes
    - 2 simulated decades in 7 to 10 days for full population
  - Some level of parallelization

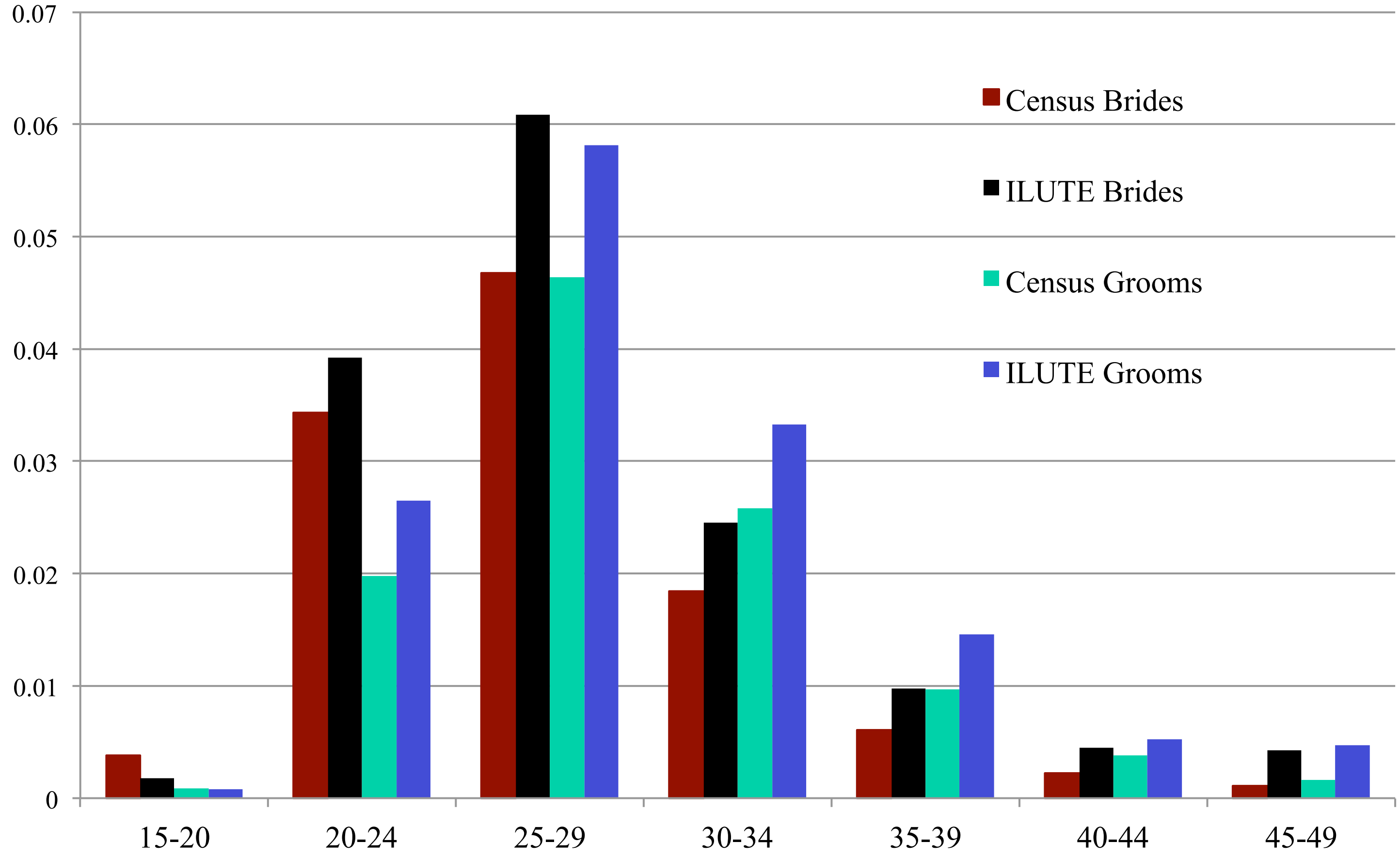
Results: Age distribution of married-individuals (2001)



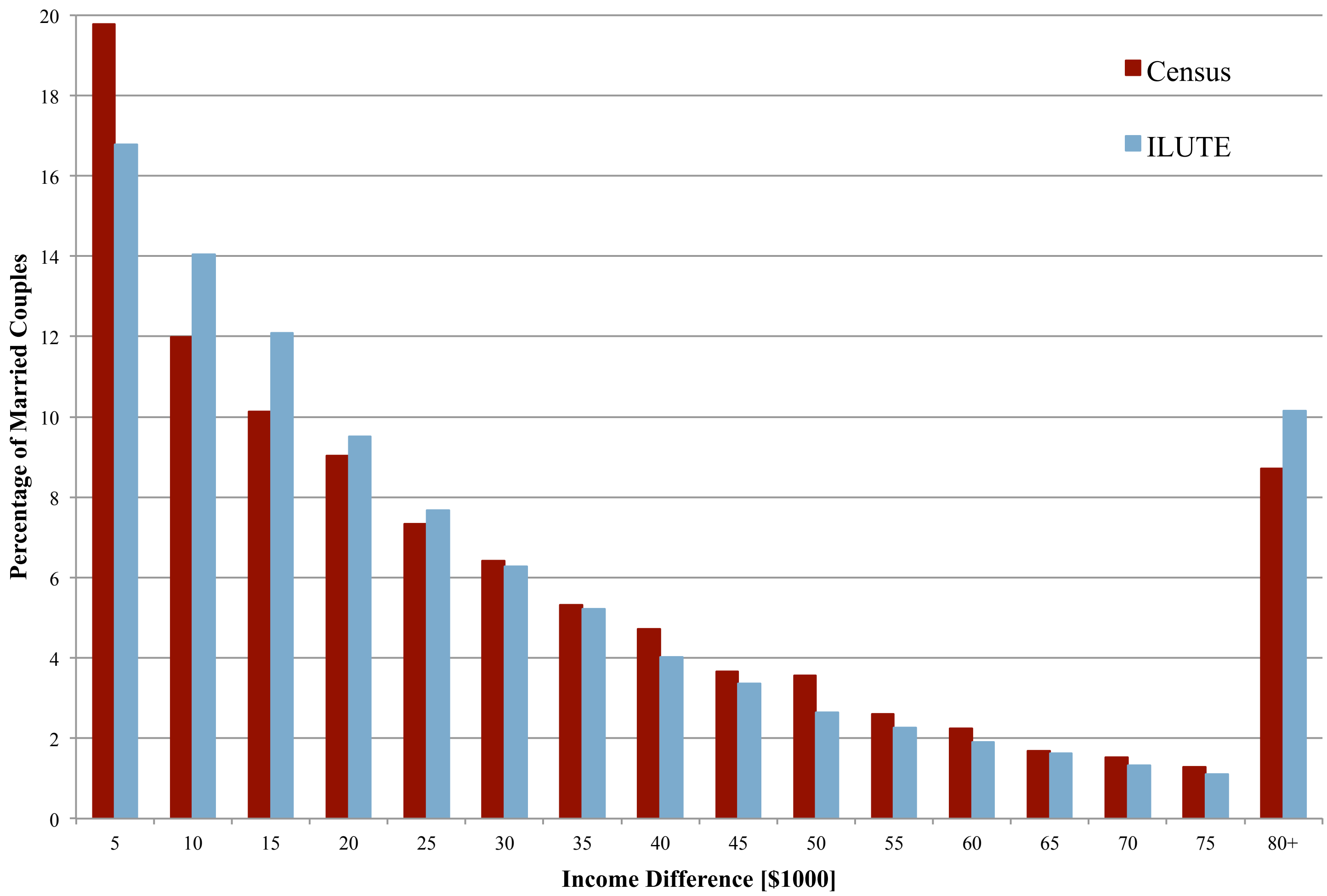
Results: Average Age by Previous Marital Status (2001)

Average Age of Newly Weds		Statistics Canada	ILUTE	% Error
Groom	Single	29.7	29.1	-2.1
	Widowed	62.4	50.9	-22.6
	Divorced	43.8	44.2	0.9
Brides	Single	27.6	27.3	-1.1
	Widowed	55.4	47.5	-16.6
	Divorced	40.3	41.7	3.4

Results: Marriage Rates by Age Group (2001)



Results: Marriage Rates Income difference (2001)



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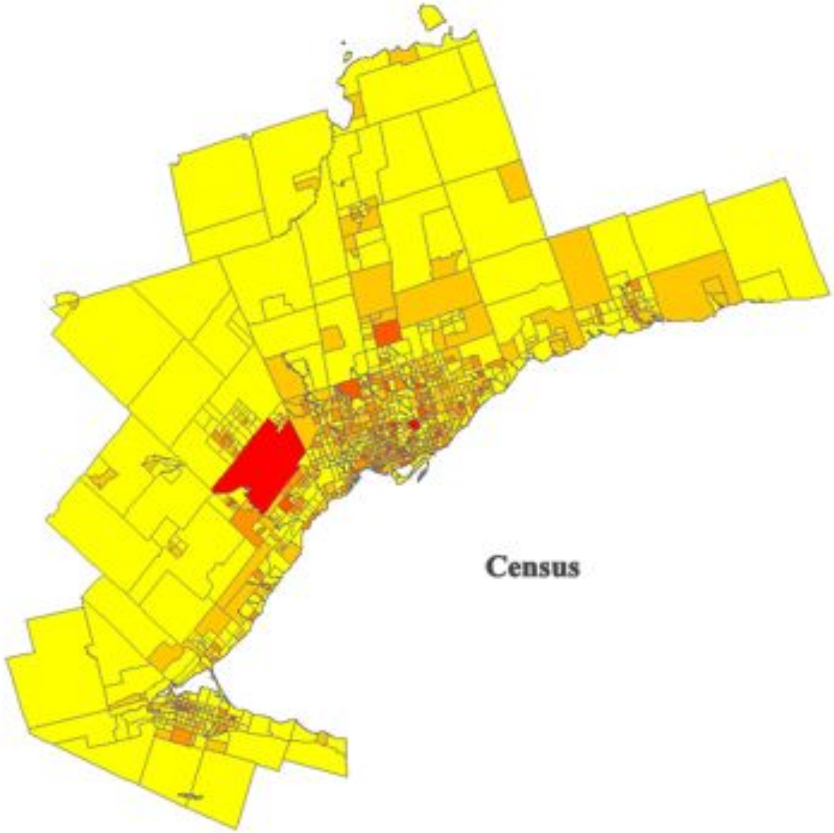
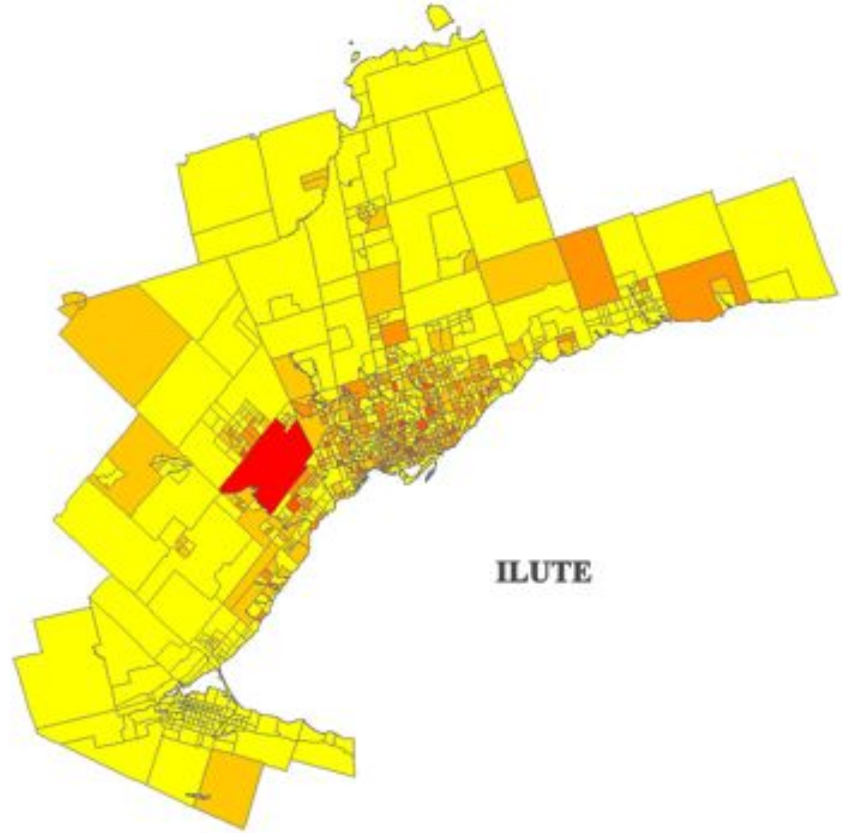
Rental Market

- Decision to rent or own
- Location decision
- Asking rent model

Results: Spatial Distribution of Rental Household (2001)





Conclusion and Future Work

- A Generalized, Graph Theoretic Solution to the Clearing Problem of Price-Taker Markets
- Applied to Marriage, and Rental within the ILUTE Modelling Framework
  - More rigorous testing
- Possible implementations of the Labour and Freight Markets
- Efficient Data Structures for Memory Management and Parallelization

Question?